# THE EFFECT OF GRAVITY ON THE PROCESS OF DRYING CONCRETE*

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An analysis is presented for the process and for the solution of a problem dealing with the distribution of moisture in a body in the diffusion drying of concrete; consideration is given to the effect of gravitation under conditions in which there is no hydration, carbonation, and aging in the material; moreover, the transport coeffients are constants.

In an external force field (such as, for example, a gravitational field) a disordered motion of the diffusion type is imposed on the ordered motion of moisture in disperse media. In the absence of gravitational forces, the transfer of mass in capillary-porous uniform structures is isotropic. In the mathematical formulation of the phenomenon we can therefore use a diffusion equation with constant coefficients. A bibliography and some of the solutions for such problems are cited in [2]. The numerical solutions in the form of nomograms for a number of moist materials and, in particular, for concrete were presented by the author at the symposium [3]. Under the considerable influence of a gravitation field, the transfer of moisture in disperse media becomes more complicated, it may be anisotropic in nature (see, for example, [14]), and in drying, the direction in which the moisture moves may prove to be significant.

In a number of papers, in the mathematical formulation of the motion of moisture in porous media, our attention was drawn to the particular importance of the term describing the action of the force of gravity [5–8]. However, in calculating the transfer of moisture through fine-pore media, and particularly in the theory of drying and in the mathematical formulation of the problem pertaining to the diffusion transfer of mass, the effect of gravity is usually neglected. It is therefore stated in the Luikov monograph [9] that the transport of moisture from a liquid or a vapor in capillary-porous colloidal structures is in the nature of diffusion. In the description of this motion, the diffusion equation contains no term for the force of gravity. Neither is the gravitational component considered in the writings of Luikov and Mikhailov [10], nor in those of Dicke, Roll, and Weber [19]. The generally held view is that the smaller the dimensions of the pores, the greater the quantity of accumulated moisture in the porous medium and the smaller the effect of gravitation. The theory of adsorption confirms the validity of this standpoint. Krischer [11] noted that the effect of gravitation could be neglected even in the drying of a bed of sand exhibiting an average particle dimension of 0.2 mm. Luikov [20] writes that with an accuracy to 6% the effect of the force of gravity is insignificant when the pores exhibit a dimension less than $10^{-3}$ cm. The effect of the force of gravity is also neglected in many engineering calculations [19].

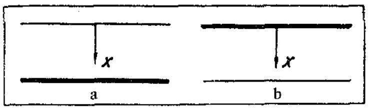


Fig. 1. Predicted scheme: a) case 1; b) case 2.

It would seem from the above-cited data that we are justified in neglecting the motion of moisture under the action of the force of gravity in the drying of concrete in which pore dimensions generally range in an interval of values from $10^3$ to 10 angstroms. The author was of this opinion until 1964. At the RILEMA symposium, Professor S. Irmei (of the Haifa Institute of Technology, Israel) raised a question about the absence of a gravitation term in the mathematical formulation of the problem on the transfer of mass in the report submitted by the author [1]. The subsequent discussion aroused interest in this question and provided the basis for the current study.

We note that soil scientists are constantly engaged in the study of gravitational effects.

Preliminary processing of the experiments carried out at our laboratory demonstrates that the effect of gravitational forces on the process of drying concrete can be measured; however, individual variations in the properties of the various specimens may significantly influence the gravitational effect.

As the basic relationship describing the transport of moisture, we employed an equation of diffusion

*The author has been engaged since 1961 at the State Institute for Technical Research (Helsinki, Finland) on the topic: "The Drying of Concrete" [1]. A brief outline of the completed research on the effect of gravitation on the process of drying concrete was published in "A Theoretical Investigation of the Effect of Gravitation on Drying of Concrete. Two Solutions of a Diffusion-Type Equation, with Consideration of the Gravitation Component" by Pihlajavaara and Rant (State Institute for Technical Research, no. 94, Helsinki, 1965). This article is a somewhat modified portion of the latter publication, including a section of the chapter entitled "Physics," a summary of the formulas from the chapter entitled "Mathematics" (without derivation of the formulas, which was the work of Dr. Rant), and certain of the nomograms from the chapter entitled "Numerical Examples."

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which took into consideration the effect of the force of gravity on the drying process and this force is naturally assumed to be independent of the body dimensions:

$$\mathbf{J} = -k \operatorname{grad} C + k_g C \mathbf{i}_g. \quad (1)$$

By introducing a new variable

$$N = \frac{C - C_e}{C_0 - C_e},$$

we can present Eq. (1) in the form

$$\mathbf{J}_N = \frac{\mathbf{J}}{C_0 - C_e} = -k \operatorname{grad} N + k_g N \mathbf{i}_g + k_g \mathbf{i}_g \frac{C_e}{C_0 - C_e}. \quad (2)$$

The last term in the right-hand member of Eq. (2) can be neglected when the vapor pressure of the moisture is in equilibrium with the partial pressure of the vapor in the ambient air and, consequently, the moisture is not in motion. If at the conclusion of the process the "equilibrium moisture" or a part of it is moved under the action of the force of gravity, the definition of $C_e$ is not entirely exact. Certain lengthy experiments have demonstrated that there is no true equilibrium state in concrete. In the following we will not consider the last term in (2).

The rate of change N, as is well known, can be represented in the following manner:

$$\frac{\partial N}{\partial h} = k \operatorname{div} \operatorname{grad} N - k_g \frac{\partial N}{\partial z}, \quad (3)$$

where k and $k_g$ are assumed constant (in the general case they may even be variable), while the z-axis coincides with the direction of the gravitational force.

With introduction of the dimensionless variables

$$x = \frac{z}{l}, \quad \alpha = \frac{k}{k_g l}, \quad \text{and} \quad t = \frac{h k_g}{l}$$

Eq. (3), for an unbounded plate, can be written as follows:

$$\frac{\partial N}{\partial t} = \alpha \frac{\partial^2 N}{\partial x^2} - \frac{\partial N}{\partial x}. \quad (4)$$

The product $\alpha t$ is the Fourier number: Fo = $\alpha t$.

In a slightly different form, Eq. (4) was first derived by T. De Kudre (Ann. der Phys., 1894). In the years following it was repeatedly used to study the various diffusion processes in the gravitational field [12–18]. It was assumed in the derivation of Eq. (1) that the drying of concrete is an isothermal process and that the transport of bound moisture under the action of gravity is insignificant; moreover, it was assumed that there is no hydration, carbonation, and aging in the drying process, and that the transport coefficients are constant. In addition, it was assumed that the concrete is a macroscopic, isotropic, and homogeneous medium.

For the remainder of this discussion it is advisable to impart a somewhat different form of notation to Eq. (1):

$$\alpha N'' - N' - \dot{N} = 0. \quad (5)$$

We will examine a horizontal infinite plate of unit thickness, assuming one of its surfaces to be moistureproof, i.e., impermeable to a flow of moisture. We are thus confronted with two problems:

1. The free surface is turned up (x = 0). If the coordinate x is reckoned in the direction of the gravitational flux (Fig. 1a), the boundary conditions are written as follows:

$$\text{when } x = 0,\ N = 0 \text{ and when } x = 1, \alpha N' - N = 0. \quad (6)$$

2. The free surface is turned down (x = 1) (Fig. 1b), so that the boundary conditions assume the form:

$$\text{when } x = 0,\ \alpha N' - N = 0 \text{ and when } x = 1\ N = 0. \quad (7)$$

The initial conditions in the two problems are the following:

$$\text{when } t = 0,\ N = 1. \quad (8)$$

In the problems under consideration we present the derived relationships for the moisture-content distribution N(x, t), the time variation in the average moisture content $\bar{N}(t) = \int_0^1 N(x, t)\,dx$, and the moisture flow through the free surface J(t) = $\alpha$N' − N.

The 1-st problem:

$$N(x, t) = \sum_{n=1}^{\infty} \frac{8\alpha^2 \lambda_n}{1 + 4\alpha^2\lambda_n^2 - 2\alpha} \left[1 \pm \frac{(-1)^n 2 \exp\left(-\frac{\alpha}{2}\right)}{\sqrt{1 + 4\alpha^2\lambda_n^2}}\right] \times \exp\left(-\omega_n t + \frac{x}{2\alpha}\right) \sin \lambda_n x; \quad (9)$$

$$\bar{N}(t) = \sum_{n=1}^{\infty} \frac{8\alpha^2 \lambda_n}{1 + 4\alpha^2\lambda_n^2 - 2\alpha} \left[1 \pm \frac{(-1)^n 2 \exp\left(-\frac{\alpha}{2}\right)}{\sqrt{1 + 4\alpha^2\lambda_n^2}}\right] \times \frac{4\alpha^2\lambda_n}{1 + 4\alpha^2\lambda_n^2} \exp(-\omega_n t); \quad (10)$$

$$J(t) = \sum_{n=1}^{\infty} \frac{8\alpha^3 \lambda_n^2}{1 + 4\alpha^2\lambda_n^2 - 2\alpha} \left[1 \pm \frac{(-1)^n 2 \exp\left(-\frac{\alpha}{2}\right)}{\sqrt{1 + 4\alpha^2\lambda_n^2}}\right] \times \exp(-\omega_n t) \quad (x = 0). \quad (11)$$

The "+" in the cited formulas pertains to $\alpha > 1/2$, while the "−" pertains to $0 < \alpha \le 1/2$. Some of the results from calculations on the basis of formula (9) are shown in Figs. 2 and 3.

In the solutions for (9)–(11) we see that $\lambda$ is a characteristic root of the equation

$$\operatorname{tg} \alpha = 2\alpha\lambda,$$

so that the trivial solution $\lambda = 0$ does not satisfy the boundary conditions.

If $\alpha > 1/2$, the first root can be defined by a trial-and-error method. The series for $\lambda_n$ has the form

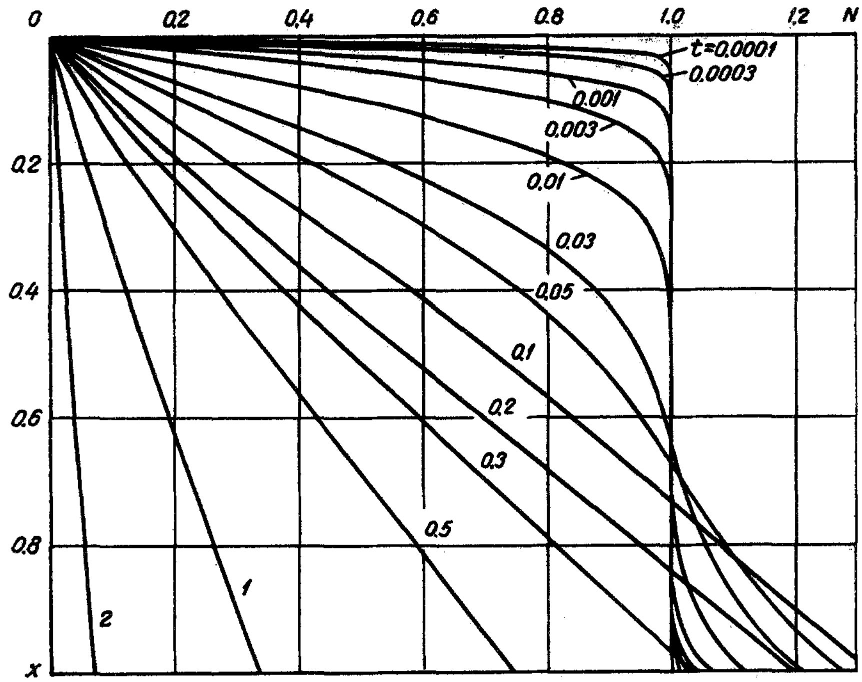


Fig. 2. Local dimensionless moisture content N of body versus dimensionless coordinate x for 1-st case at various values of dimensionless time t ($\alpha = 1$).

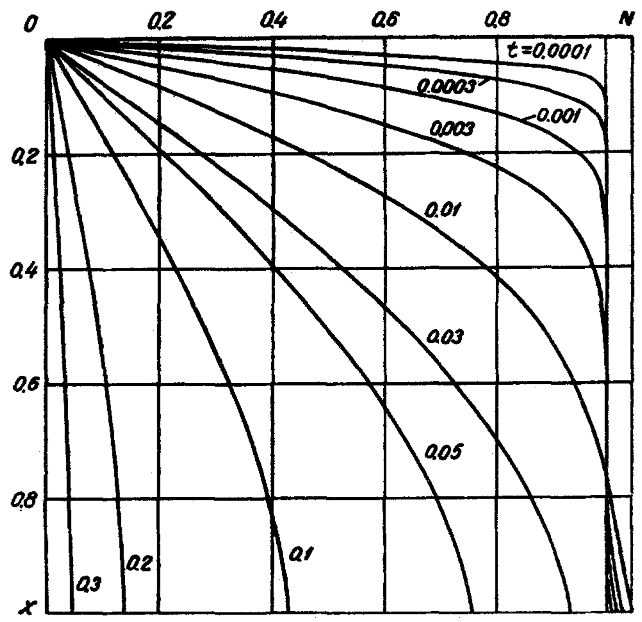


Fig. 3. Local dimensionless moisture content N of body versus dimensionless coordinate x for 1-st case at various values of dimensionless time t ($\alpha = 5$).

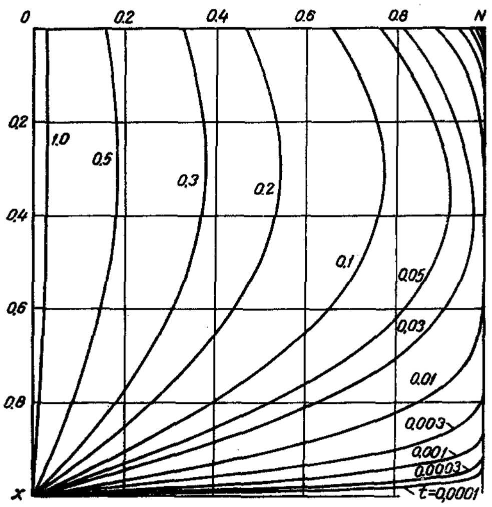


Fig. 4. Local dimensionless moisture content N of body versus dimensionless coordinate x for 2-nd case at various values of dimensionless time t ($\alpha = 1$).

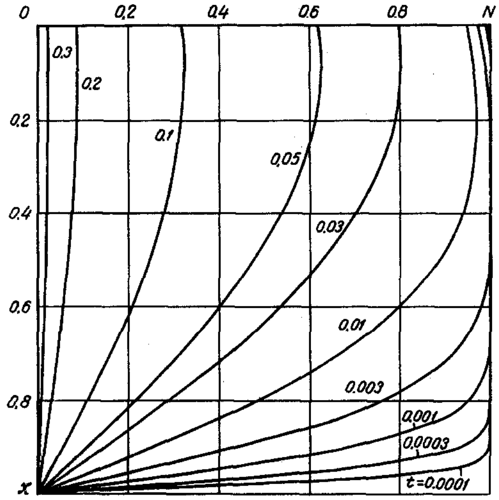


Fig. 5. Local dimensionless moisture content N of body versus dimensionless coordinate x for 2-nd case at various values of dimensionless time t ($\alpha = 5$).

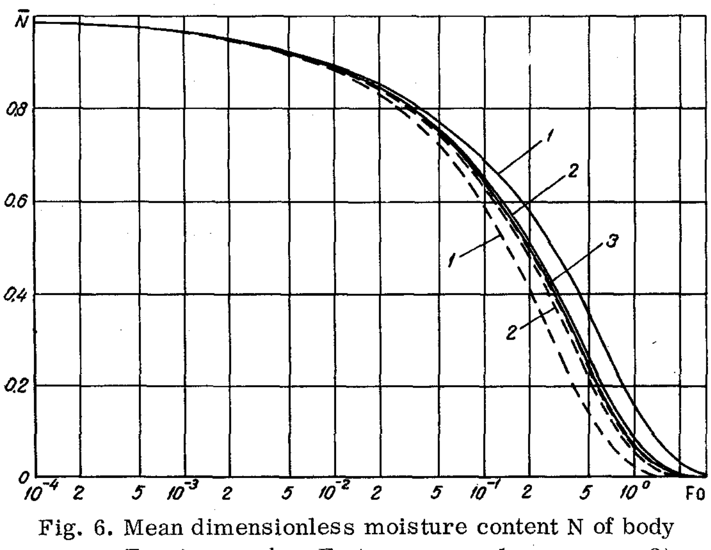


Fig. 6. Mean dimensionless moisture content N of body versus Fourier number Fo (—— case 1; - - - case 2); 1) $\alpha = 1$; 2) $\alpha = 5$; 3) $\alpha = 50$.

$$\lambda_n = \varphi_n - \frac{1}{2\alpha}\varphi_n^{-1} - \left(\frac{1}{4\alpha^2} - \frac{1}{24\alpha^3}\right)\varphi_n^{-3} - \\ - \left(\frac{1}{4\alpha^3} - \frac{1}{12\alpha^4} + \frac{1}{160\alpha^5}\right)\varphi_n^{-5},$$

where

$$\left.\begin{aligned}\varphi_n &= (2n+1)\pi/2 \text{ when } 0<\alpha \leqslant 1/2 \\ \varphi_n &= (2n-1)\pi/2 \text{ when } \alpha > 1/2\end{aligned}\right\} n = 1, 2, 3, \ldots$$

The error in the determination of $\lambda_n$ is equal to

$$|\Delta\lambda_n| = O(\alpha^{-4}\varphi_n^{-7}) \text{ when } \alpha \gg 1.$$

The constant $\omega_n$ is related to $\lambda_n$ by the equation

$$\omega_n = (1 + 4\alpha^2\lambda_n^2)/4\alpha. \tag{12}$$

The 2-nd problem:

$$N(x,t) = \\ = \sum_{n=1}^{\infty} \frac{8\alpha^2\lambda_n}{1+4\alpha^2\lambda_n^2+2\alpha}\left[1 - \frac{(-1)^n 2\exp(\alpha/2)}{\sqrt{1+4\alpha^2\lambda_n^2}}\right] \times \\ \times \exp\left(-\omega_n t - \frac{1-x}{2\alpha}\right) \sin\lambda_n(1-x); \tag{13}$$

$$\bar{N}(t) = \\ = \sum_{n=1}^{\infty} \frac{8\alpha^2\lambda_n}{1+4\alpha^2\lambda_n^2+2\alpha}\left[1 - \frac{(-1)^n 2\exp(\alpha/2)}{\sqrt{1+4\alpha^2\lambda_n^2}}\right] \times \\ \times \frac{4\alpha^2\lambda_n}{1+4\alpha^2\lambda_n^2} \exp(-\omega_n t); \tag{14}$$

$$J(t) = \\ = -\sum_{n=1}^{\infty} \frac{8\alpha^3\lambda_n^2}{1+4\alpha^2\lambda_n^2+2\alpha}\left[1 - \frac{(-1)^n 2\exp(\alpha/2)}{\sqrt{1+4\alpha^2\lambda_n^2}}\right] \times \\ \times \exp(-\omega_n t) \quad (x=1). \tag{15}$$

Some of the results from our calculations according to formula (13) are shown in Figs. 4 and 5.

The positive roots of the solutions for (13)–(15) are the characteristic roots of the equation

$$\operatorname{tg}\lambda = -2\alpha\lambda.$$

Here $\lambda = 0$ also fails to satisfy the conditions. We can determine $\lambda_n$ from the approximate equation

$$\lambda_n = \varphi_n + \frac{1}{2\alpha}\varphi_n^{-1} - \left(\frac{1}{4\alpha^2} + \frac{1}{24\alpha^3}\right)\varphi_n^{-3} + \\ + \left(\frac{1}{4\alpha^3} + \frac{1}{12\alpha^4} + \frac{1}{160\alpha^5}\right)\varphi_n^{-5},$$

where $\varphi_n = (2n-1)\pi/2$, n = 1, 2, 3, ... The error in the determination of $\lambda_n$ is of the same order as in the previous problem, with the constant $\omega_n$ determined from relationship (12). The calculations on the basis of the above-cited solutions were carried out on an Elliot 803A digital computer. Some of the results from the calculation according to formulas (10) and (14) are shown in Fig. 6.

Figure 7 shows the experimental curves for the drying of moist plates (7.4 cm in diameter and 1.5 cm in thickness) made from a standard cement solution

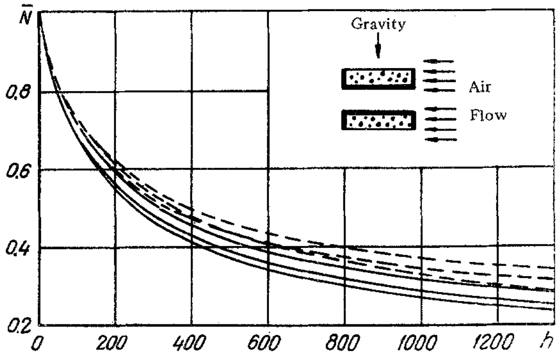


Fig. 7. Mean dimensionless moisture content $\bar{N}$ of body versus time h; v = 5–7 m/sec; $\varphi$ = = 40%; T = 20° C; (for notation, see Fig. 6).

(the cement-to-water ratio is 0.56; the cement mass to solid-phase ratio is equal to 1 : 1.2; the maximum dimension for the standard solid-phase particles (quartz) is equal to 1.4 mm; the average force of compression on a specimen 4 cm$^3$ in volume is $4.07 \cdot 10^7$ N/m$^2$). The preliminary experiments on the effect of the force of gravity were carried out by the author over a period of eight weeks. A diagram of the experiment is shown in the upper part of the figure. The plates were kept in a wind tunnel in a horizontal position during the course of the experiment to eliminate natural convection. The experiment was begun precisely at the instant at which the vapor pressure of the surface moisture of the specimen was in equilibrium with the partial vapor pressure of the ambient air.

The curves in Fig. 7 yield the following average approximate values: $k = 2 \cdot 10^{-11}$ m$^2$/sec; $k_g = 7 \cdot 10^{-10}$ m/sec; $\alpha = 2$.

In conclusion, we note that these experiments are only preliminary and the derived results are in need of further refinement.

## NOTATION

C is the moisture content (concentration of free water and steam in concrete), kg/m$^3$; $C_0$ is the initial moisture content, kg/m$^3$; $C_e$ is the equilibrium moisture content, kg/m$^3$; $\bar{C}$ is the moisture content averaged with respect to material volume, kg/m$^3$; Fo is Fourier number; h is the time, sec; $i_g$ is the unit vector in the direction of gravitation; J is the specific moisture flow, kg/m$^2$sec; $J_N$ is the linear velocity of molecular moisture flow, m/sec; J(x, t) is the dimensionless moisture flow; k is the moisture conductivity or diffusivity coefficient, m$^2$/sec; $k_g$ is the gravitational "mobility" of moisture, m/sec; $l$ is the characteristic dimension (here, thickness of plate), m; N is the local dimensionless moisture content; $\bar{N}$ is the mean dimensionless moisture content in the body; 0( ) is the order of the quantity; t is the dimensionless time; T is the temperature, ° C; x is the dimensionless coordinate; v is the velocity, m/sec; z is the coordinate in the gravitation direction, m; $\alpha$ is the dimensionless parameter; $\Delta$ is the error; $\varphi_n$ is the odd multiplier; $\varphi$ is the relative air humidity, %; ' denotes the x derivative; · denotes the t derivative.

REFERENCES

1. S. E. Pihlajavaara, "Notes on the drying of concrete," State Institute for Technical Research, Report, Series III, Helsinki, 1963.

2. S. E. Pihlajavaara, Instructory Bibliography for Research on Drying of Concrete, 2-nd ed., State Institute for Technical Research, Laboratory of Concrete Technology, Helsinki, 1964.

3. S. E. Pihlajavaara, "Estimation of the drying of concrete," RILEMA Symposium on Structural Moisture, Helsinki, August, 1965.

4. W. R. Gardner, "Mathematics of isothermal water conduction in unsaturated soils," Highway Res. Board Special Rep. 40, Washington, D. C., 1958.

5. J. R. Philip and D. A. De Vries, Trans. Amer. Geophys. Union, vol. 38, no. 2, 1957.

6. D. A. De Vrier, Trans. Amer. Geophys. Union, vol. 39, no. 5, 1958.

7. J. P. Philips, "Physics of water movement in porous solids," Highway Res. Board, Spec. Rep. 40, Washington, D. C., 1958.

8. D. A. De Vries, Ing. [in Dutch], 5, 1962.

9. A. W. Luikov, Experimentalle und Theoretische Grundlagen der Trocknung. VEB Verlag Technik, Berlin, p. 140, 1955.

10. A. V. Luikov and Y. A. Mikhailov, Theory of Energy and Mass Transfer, Prentice-Hall Inc., Englewood Cliffs, N. J. 1961.

11. O. Krischer, Die Wissenschaftliche Grundlagen der Trocknungstechnik, Springer Verlag, Berlin, 1963.

12. M. Mason and W. Weaver, Phys. Rev., 23, 3, 1924.

13. R. Fürth, Zs. Physik, 40, 5, 1926.

14. R. Fürth, B. Zuber, and K. V. Sittle, Z. f. Physik, **79**, 5–6, 1932.

15. R. Fürth, Sedimentation und Brownische Bewegung im Schwerefeld, in: Frank and Mises, Differentialgleichungen der Physik, Braunschweig, 1935.

16. W. Jost, Diffusion in Solids, Liquids, Gases, Academic Press. Inc., New York, 1960.

17. R. C. L. Bosworth, Transport Processes in Applied Chemistry, Sir Isaac Pitman and Sons, Ltd., Sydney, 1956.

18. H. Fujita, Mathematical Theory of Sedimentation Analysis, Academic Press, New York and London, 1962.

19. R. H. Dicke, Roll, and J. Weber, "Gravity experiments," Int. Science and Technology, 5, 1964.

20. A. V. Luikov, Transporterscheinungen in kapillarporösen Körpern, Akademik Verlag, Berlin, 1958.

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